EE2003 Circuit Theory

Chapter 7

First-Order Circuits

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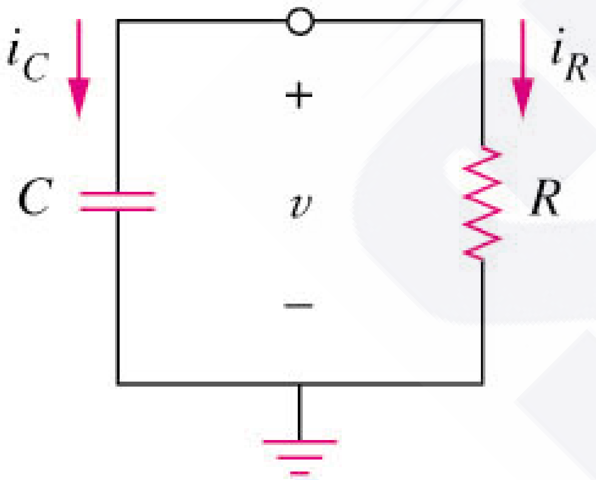
7.1 The Source-Free RC Circuit
7.2 The Source-Free RL Circuit
7.3 Unit-step Function
7.4 Step Response of an RC Circuit
7.5 Step Response of an RL Circuit

7.1 The Source-Free RC Circuit (1)

- A **first-order circuit** is characterized by a first-order differential equation.



By KCL

$$i_R + i_C = 0 \rightarrow \frac{v}{R} + C\frac{dv}{dt} = 0$$

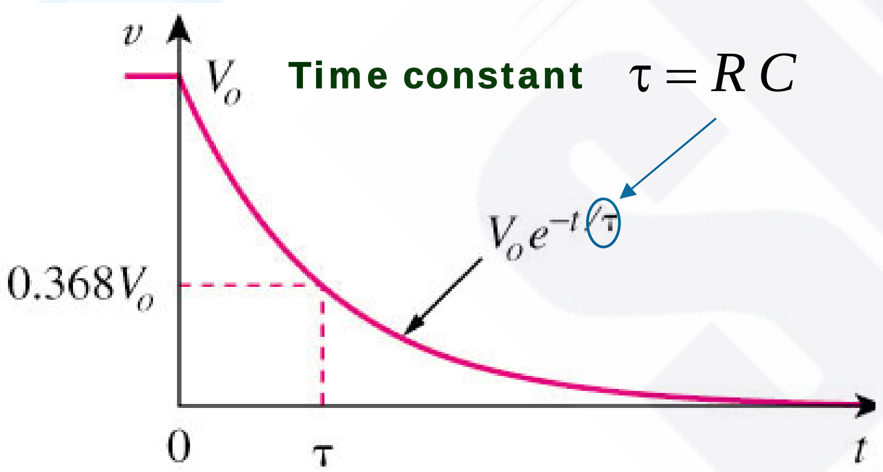
Ohms law

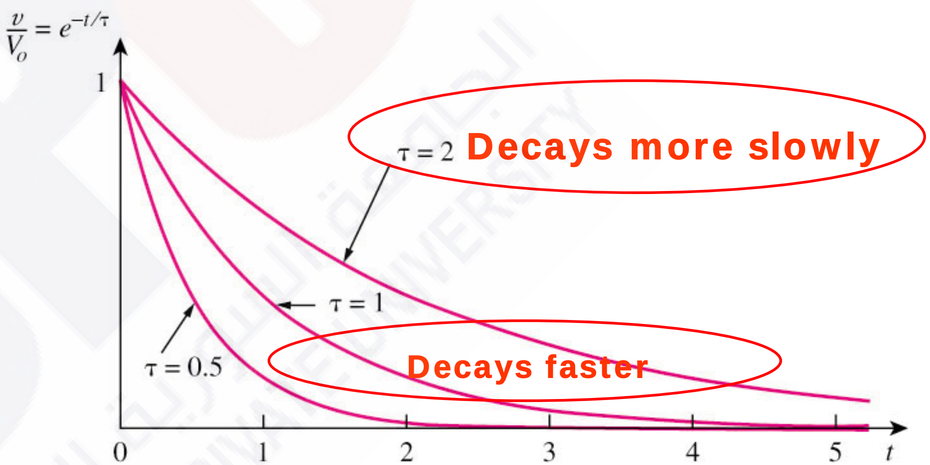
Capacitor law

- Apply Kirchhoff's laws to purely resistive circuit results in algebraic equations.
- Apply the laws to RC and RL circuits produces differential equations.

7.1 The Source-Free RC Circuit (2)

- The **natural response** of a circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.



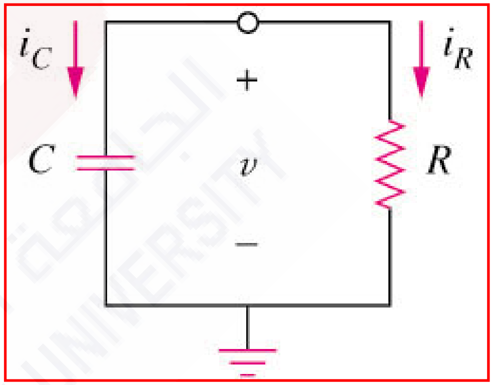


- The **time constant** τ of a circuit is the time required for the response to decay by a factor of 1/*e* or 36.8% of its initial value.
- v decays faster for small τ and slower for large τ.

7.1 The Source-Free RC Circuit (3)

The key to working with a source-free RC circuit is finding:

$$v(t) = V_0 \, e^{-t/\tau}$$ where $$\tau = RC$$



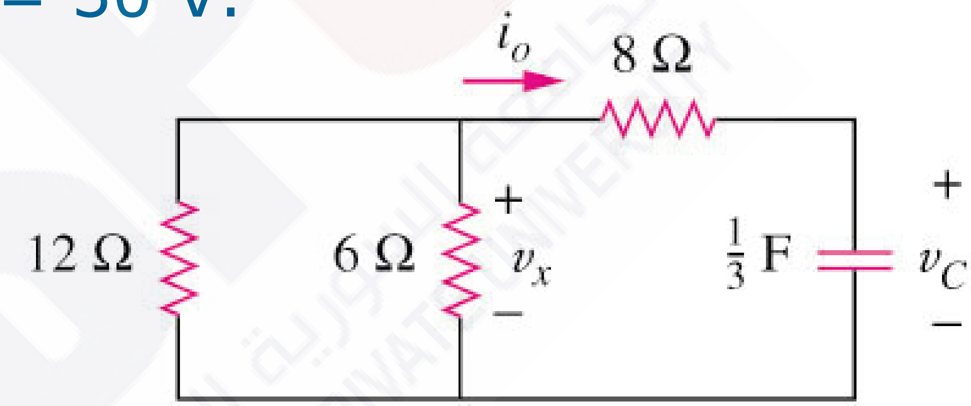
1. The initial voltage $v(0) = V_0$ across the capacitor.
2. The time constant $\tau = RC$.

7.1 The Source-Free RC Circuit (4)

Example 1

Refer to the circuit below, determine v_C, v_x, and i_o for $t \geq 0$.

Assume that $v_C(0) = 30$ V.



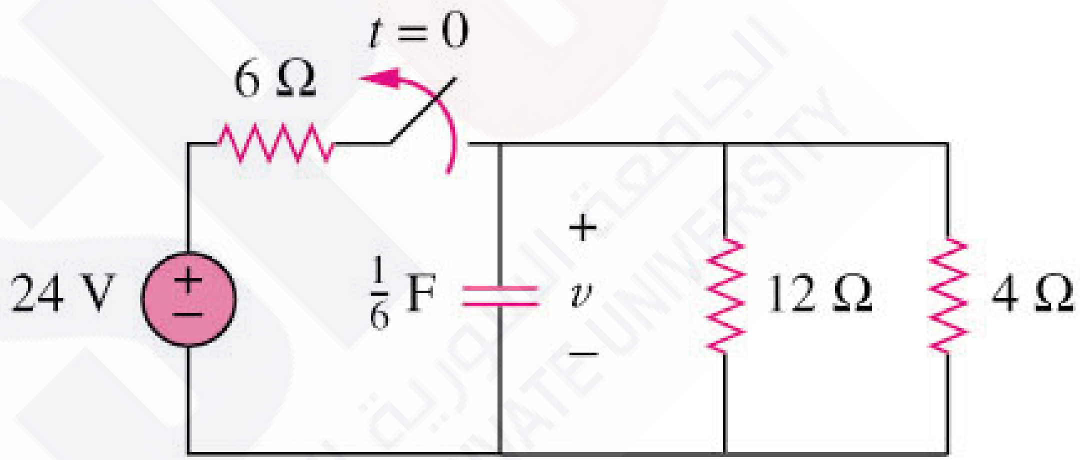
- Please refer to lecture or textbook for more detail elaboration.

Answer: $v_C = 30e^{-0.25t}$ V ; $v_x = 10e^{-0.25t}$; $i_o = -2.5e^{-0.25t}$ A

7.1 The Source-Free RC Circuit (5)

Example 2

The switch in circuit below is opened at t = 0, find v(t) for t ≥ 0.

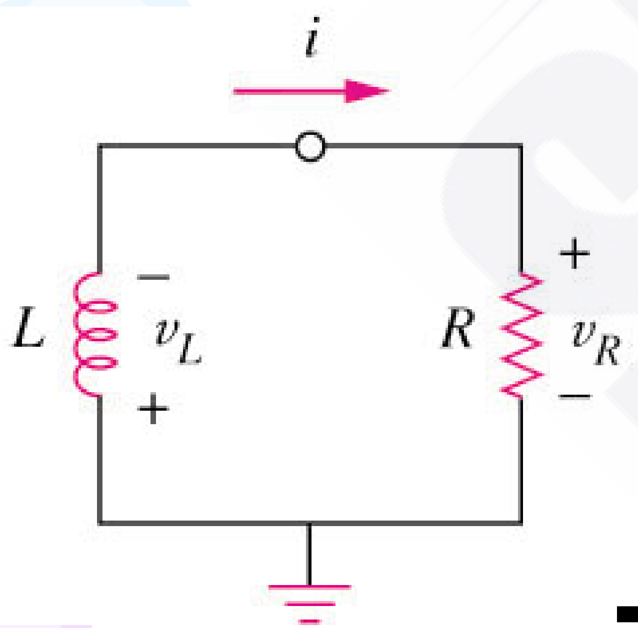


- Please refer to lecture or textbook for more detail elaboration.

Answer: $V(t) = 8e^{-2t}$ V

7.2 The Source-Free RL Circuit (1)

- A **first-order RL circuit** consists of a inductor L (or its equivalent) and a resistor (or its equivalent)



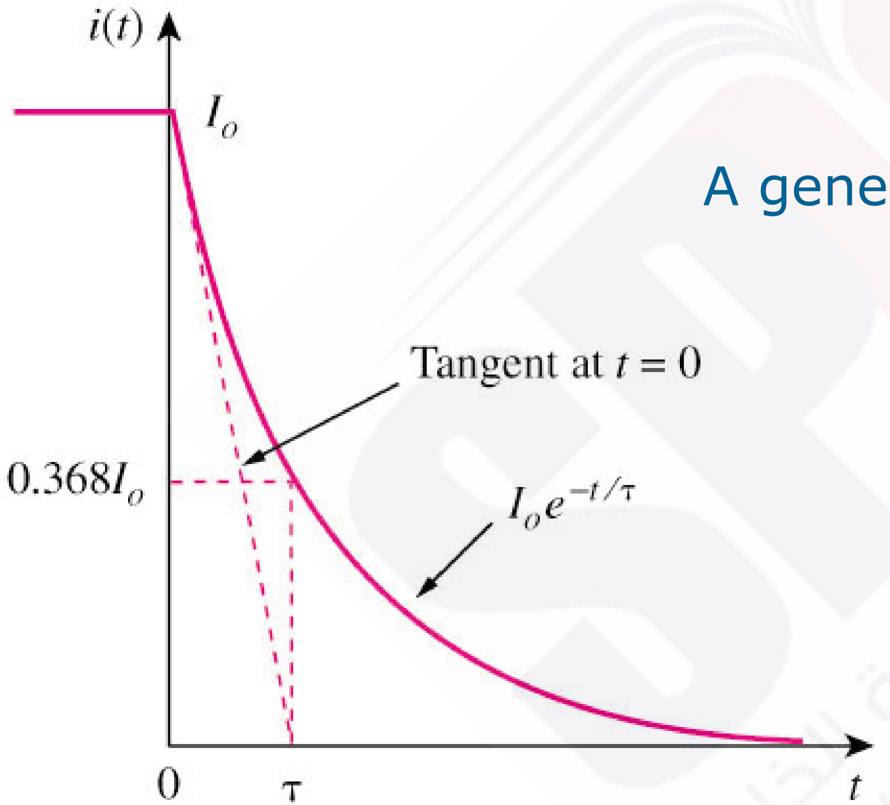
By KVL $$v_L + v_R = 0$$

→ $$L\frac{di}{dt} + iR = 0$$

Inductors law ($L\frac{di}{dt}$)   Ohms law (iR)

→ $$\frac{di}{i} = -\frac{R}{L}dt$$ → $$i(t) = I_0\, e^{-Rt/L}$$

7.2 The Source-Free RL Circuit (2)



A general form representing a RL

$$i(t) = I_0 \, e^{-t/\tau}$$

where $\tau = \dfrac{L}{R}$

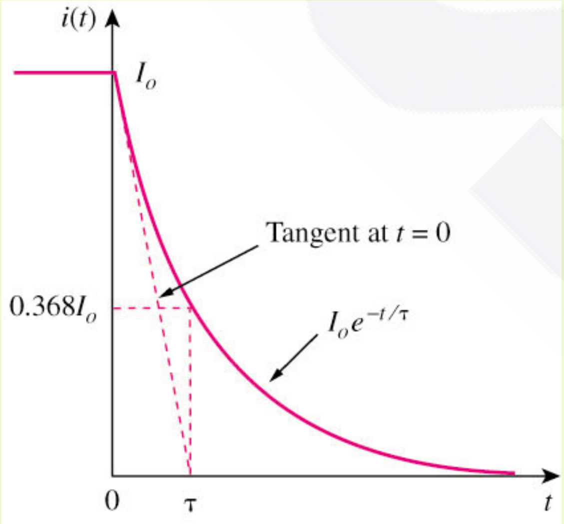
- The **time constant** τ of a circuit is the time required for the response to decay by a factor of 1/*e* or 36.8% of its initial value.
- $i(t)$ decays faster for small τ and slower for large τ.
- The general form is very similar to a RC source-free circuit.

7.2 The Source-Free RL Circuit (3)

Comparison between a RL and RC circuit

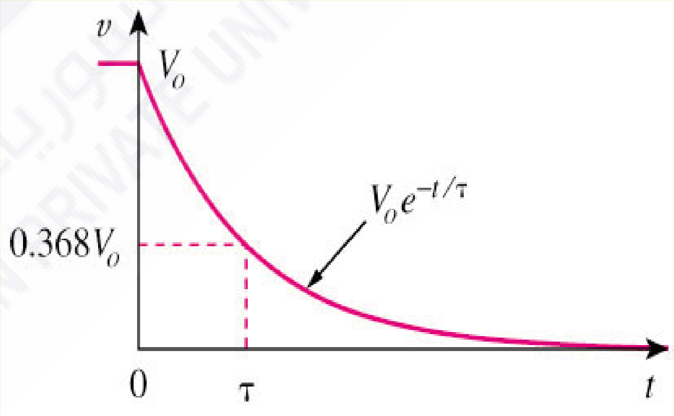
A RL source-free circuit

$$i(t) = I_0 \, e^{-t/\tau}$$ **where** $$\tau = \frac{L}{R}$$



A RC source-free circuit

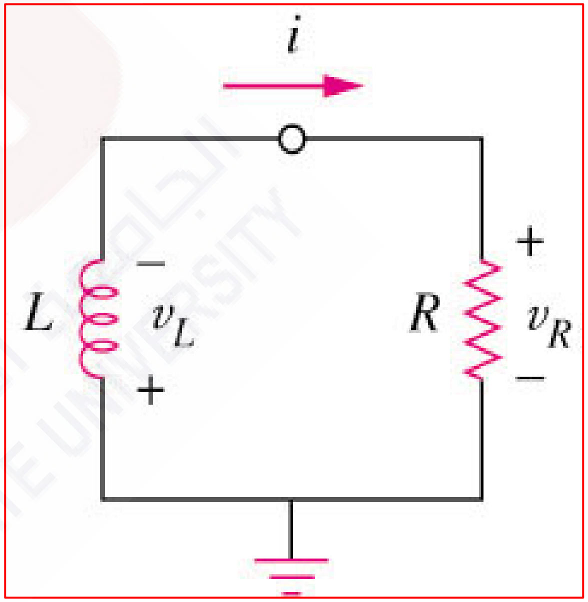
$$v(t) = V_0 \, e^{-t/\tau}$$ **where** $$\tau = RC$$



7.2 The Source-Free RL Circuit (4)

The key to working with a source-free RL circuit is finding:

$$i(t) = I_0 \, e^{-t/\tau}$$ where $$\tau = \frac{L}{R}$$



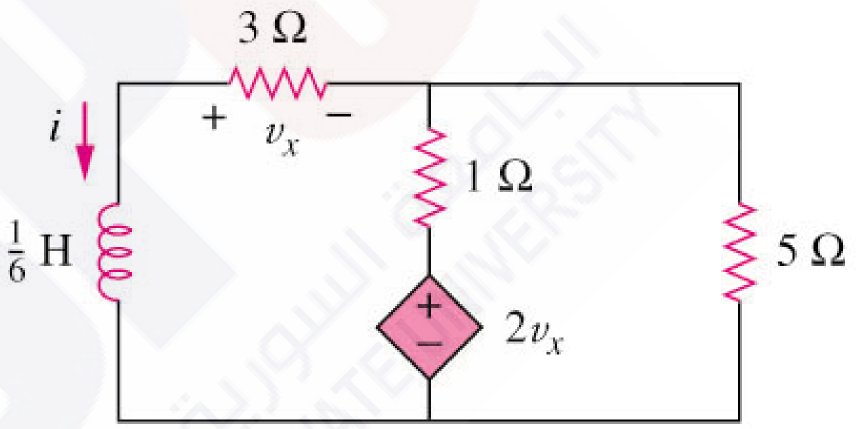
1. The initial voltage $i(0) = I_0$ through the inductor.
2. The time constant τ = L/R.

7.2 The Source-Free RL Circuit (5)

Example 3

Find i and v_x in the circuit.

Assume that i(0) = 5 A.



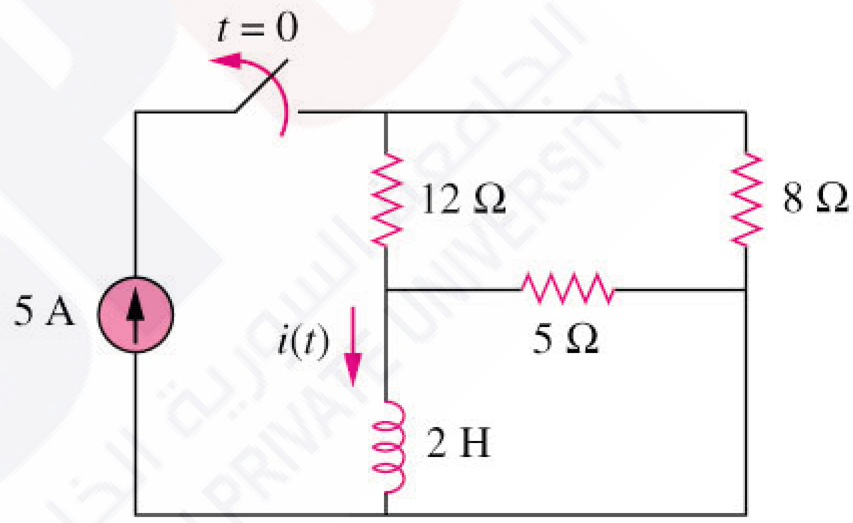
- Please refer to lecture or textbook for more detail elaboration.

Answer: $i(t) = 5e^{-53t}$ A

7.2 The Source-Free RL Circuit (6)

Example 4

For the circuit, find i(t) for t > 0.



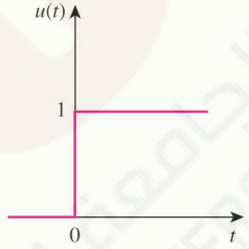
- Please refer to lecture or textbook for more detail elaboration.

Answer: $i(t) = 2e^{-2t}$ A

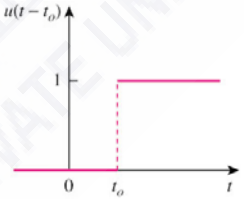
7.3 Unit-Step Function (1)

- The **unit step function** $u(t)$ is 0 for negative values of t and 1 for positive values of t.

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

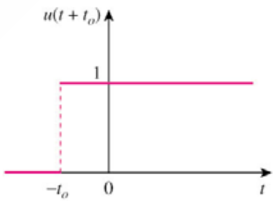


$$u(t - t_o) = \begin{cases} 0, & t < t_o \\ 1, & t > t_o \end{cases}$$



(a)

$$u(t + t_o) = \begin{cases} 0, & t < -t_o \\ 1, & t > -t_o \end{cases}$$

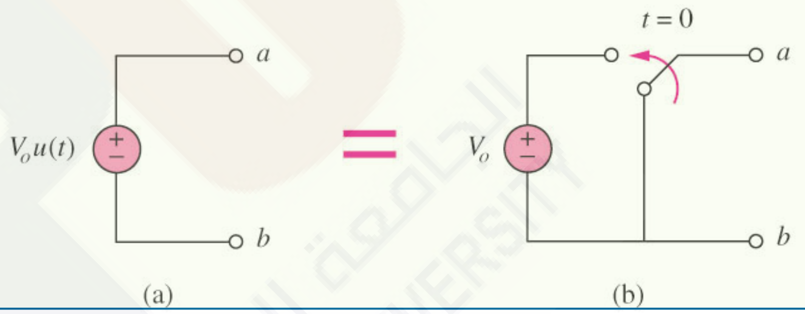


(b)

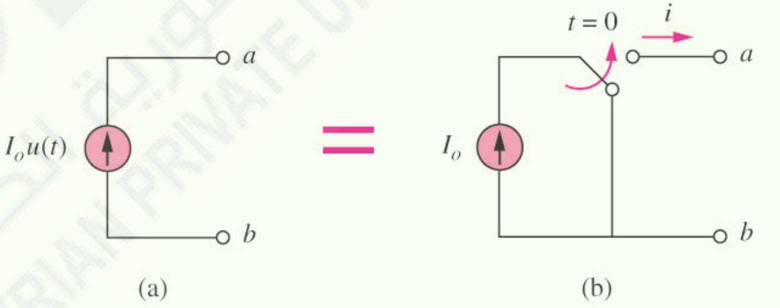
7.3 Unit-Step Function (2)

Represent an abrupt change for:

1. voltage source.

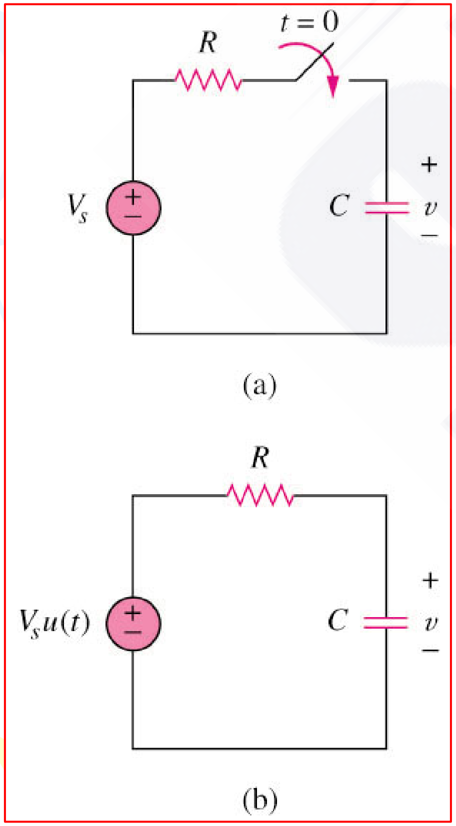


2. for current source:



7.4 The Step-Response of a RC Circuit (1)

- The **step response** of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.



- **Initial condition:**

  $v(0-) = v(0+) = V_0$

- Applying KCL,

$$c\frac{dv}{dt}+\frac{v-V_s u(t)}{R}=0$$

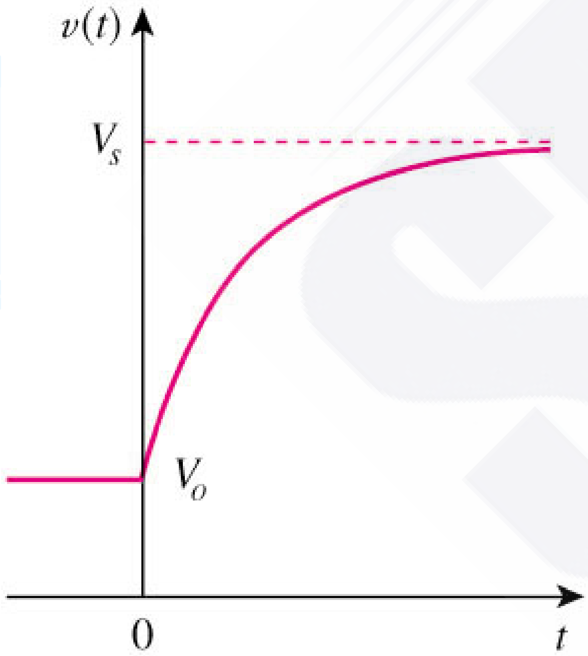
or

$$\frac{dv}{dt}=-\frac{v-V_s}{RC}u(t)$$

- Where u(t) is the unit-step function

7.4 The Step-Response of a RC Circuit (2)

- Integrating both sides and considering the initial conditions, the solution of the equation is:



$$v(t) = \begin{cases} V_0 & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

Final value at t -> ∞

Initial value at t = 0

Source-free Response

Complete Response = Natural response (stored energy) + Forced Response (independent source)

$= V_0 e^{-t/\tau} + V_s(1 - e^{-t/\tau})$

7.4 The Step-Response of a RC Circuit (3)

Three steps to find out the step response of an RC circuit:

1. The initial capacitor voltage v(0).
2. The final capacitor voltage v(∞) — DC voltage across C.
3. The time constant τ.

$$v(t) = v(\infty) + [v(0+) - v(\infty)]e^{-t/\tau}$$

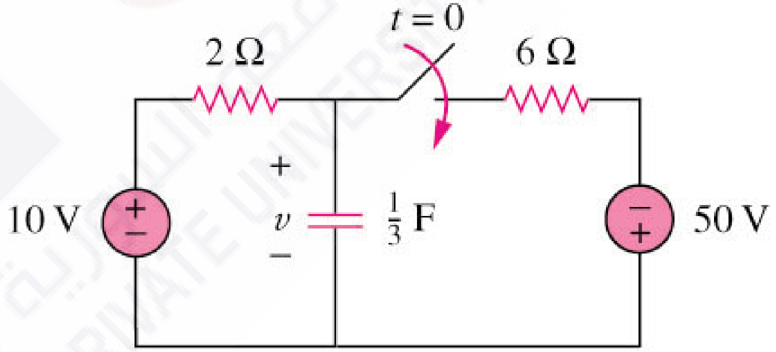
Note: The above method is a short-cut method. You may also determine the solution by setting up the circuit formula directly using KCL, KVL , ohms law, capacitor and inductor VI laws.

7.4 The Step-Response of a RC Circuit (4)

Example 5

Find v(t) for t > 0 in the circuit in below. Assume the switch has been open for a long time and is closed at t = 0.

Calculate v(t) at t = 0.5.

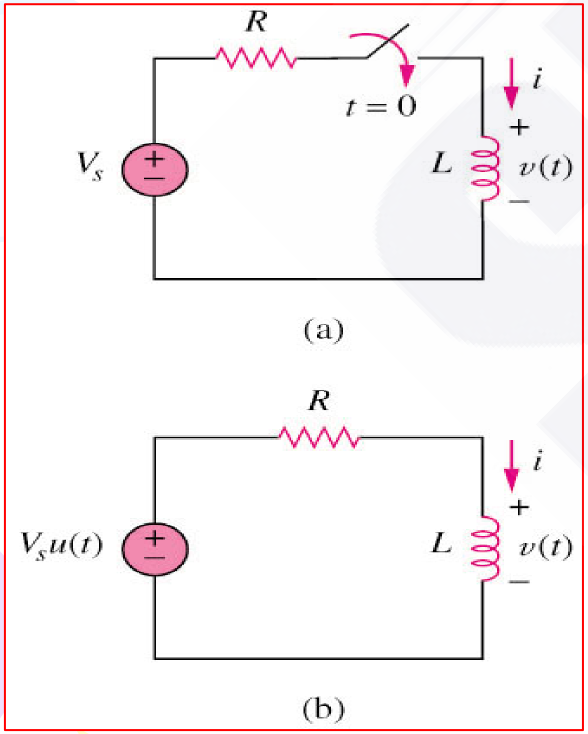


- Please refer to lecture or textbook for more detail elaboration.

Answer: $v(t) = 15e^{-2t} - 5$ *and v(0.5) = 0.5182V*

7.5 The Step-response of a RL Circuit (1)

- The **step response** of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.



- **Initial current**
  $i(0-) = i(0+) = I_o$
- **Final inductor current**
  $i(\infty) = V_s/R$
- Time constant $\tau = L/R$

$$i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}} u(t)$$

7.5 The Step-Response of a RL Circuit (2)

Three steps to find out the step response of an RL circuit:

1. The initial inductor current i(0) at t = 0+.
2. The final inductor current i(∞).
3. The time constant τ.

$$i(t) = i(\infty) + [i(0+) - i(\infty)]\, e^{-t/\tau}$$

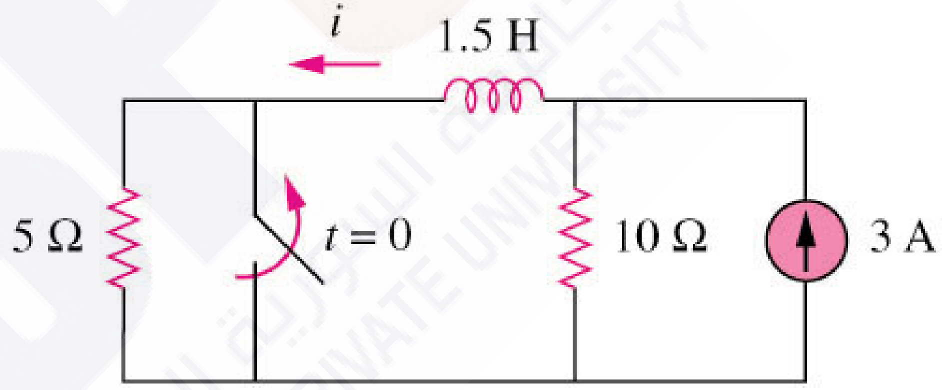
Note: The above method is a short-cut method. You may also determine the solution by setting up the circuit formula directly using KCL, KVL , ohms law, capacitor and inductor VI laws.

7.5 The Step-Response of a RL Circuit (4)

Example 6

The switch in the circuit shown below has been closed for a long time. It opens at t = 0.

Find i(t) for t > 0.



- Please refer to lecture or textbook for more detail elaboration.

Answer: $i(t) = 2 + e^{-10t}$